

IFT-11-00  
 TOKUSHIMA Report  
 March, 2000

## Signals of CP Violation in Distributions of $t\bar{t}$ Decay Products at Linear Colliders

BOHDAN GRZADKOWSKI<sup>1), a)</sup> and ZENRŌ HIOKI<sup>2), b)</sup>

1) *Institute of Theoretical Physics, Warsaw University  
 Hoża 69, PL-00-681 Warsaw, POLAND*

2) *Institute of Theoretical Physics, University of Tokushima  
 Tokushima 770-8502, JAPAN*

### ABSTRACT

Angular and energy distributions for leptons and bottom quarks in the process  $e^+e^- \rightarrow t\bar{t} \rightarrow \ell^\pm/\overset{\scriptscriptstyle(-)}{b} \cdots$  have been calculated assuming the most general top-quark couplings. The double distributions depend both on modification of the  $t\bar{t}$  production and  $\overset{\scriptscriptstyle(-)}{t} \rightarrow \overset{\scriptscriptstyle(-)}{b} W$  decay vertices. However, the leptonic angular distribution turned out to be insensitive to non-standard parts of  $Wtb$  vertex. The method of optimal observables have been used to estimate sensitivity of future measurements at linear  $e^+e^-$  colliders to top-quark couplings.

PACS: 13.65.+i

Keywords: top quark, CP violation, anomalous top-quark interactions

---

<sup>a)</sup>E-mail address: bohdan.grzadkowski@fuw.edu.pl

<sup>b)</sup>E-mail address: hioki@ias.tokushima-u.ac.jp

# 1 Introduction

In spite of the fact that the top quark has been discovered already several years ago[1] its interactions are still unknown. Therefore it remains an open question if the top-quark couplings obey the Standard Model (SM) scheme of the electroweak forces or there exists a contribution from physics beyond the SM. In this talk<sup>#1</sup> I will try to use angular and energy distributions of top-quark decay products in the process  $e^+e^- \rightarrow t\bar{t} \rightarrow \ell^\pm/\overset{\leftarrow}{b} \cdots$  in order to estimate how precisely top quark couplings could be determined at future linear collider.

We will parameterize  $t\bar{t}$  couplings to the photon and the  $Z$  boson in the following way

$$\Gamma_{vt\bar{t}}^\mu = \frac{g}{2} \bar{u}(p_t) \left[ \gamma^\mu \{ A_v + \delta A_v - (B_v + \delta B_v) \gamma_5 \} + \frac{(p_t - p_{\bar{t}})^\mu}{2m_t} (\delta C_v - \delta D_v \gamma_5) \right] v(p_{\bar{t}}), \quad (1)$$

where  $g$  denotes the  $SU(2)$  gauge coupling constant,  $v = \gamma, Z$ , and

$$A_\gamma = \frac{4}{3} \sin \theta_W, \quad B_\gamma = 0, \quad A_Z = \frac{1}{2 \cos \theta_W} \left( 1 - \frac{8}{3} \sin^2 \theta_W \right), \quad B_Z = \frac{1}{2 \cos \theta_W}$$

denote the SM contributions to the vertices. Among the above non-SM form factors,  $\delta A_{\gamma,Z}$ ,  $\delta B_{\gamma,Z}$ ,  $\delta C_{\gamma,Z}$  describe  $CP$ -conserving while  $\delta D_{\gamma,Z}$  parameterizes  $CP$ -violating interactions.

Similarly, we will adopt the following parameterization of the  $Wtb$  vertex suitable for the  $t$  and  $\bar{t}$  decays:

$$\begin{aligned} \Gamma_{Wtb}^\mu &= -\frac{g}{\sqrt{2}} V_{tb} \bar{u}(p_b) \left[ \gamma^\mu (f_1^L P_L + f_1^R P_R) - \frac{i\sigma^{\mu\nu} k_\nu}{M_W} (f_2^L P_L + f_2^R P_R) \right] u(p_t), \\ \bar{\Gamma}_{Wtb}^\mu &= -\frac{g}{\sqrt{2}} V_{tb}^* \bar{v}(p_{\bar{t}}) \left[ \gamma^\mu (\bar{f}_1^L P_L + \bar{f}_1^R P_R) - \frac{i\sigma^{\mu\nu} k_\nu}{M_W} (\bar{f}_2^L P_L + \bar{f}_2^R P_R) \right] v(p_{\bar{b}}), \end{aligned} \quad (2)$$

where  $P_{L/R} = (1 \mp \gamma_5)/2$ ,  $V_{tb}$  is the  $(tb)$  element of the Kobayashi-Maskawa matrix and  $k$  is the momentum of  $W$ .

It will be assumed here that interactions of leptons with gauge bosons are properly described by the SM. Through the calculations all fermions except the top quark will be considered as massless. We will also neglect terms quadratic in non-standard form factors.

---

<sup>#1</sup>Presented by B. Grzadkowski

## 2 Angular and Energy Distributions

In this section we will present results for  $d^2\sigma/dx_f d \cos \theta_f$  of the top-quark decay product  $f$ , where  $f$  could be either  $\ell^\pm$  or  $\overset{(-)}{b}$ ,  $x_f$  denotes the normalized energy of  $f$  and  $\theta_f$  is the angle between the  $e^-$  beam direction and the direction of  $f$  momentum in the  $e^+e^-$  CM frame.

Using the technique developed by Kawasaki, Shirafuji and Tsai[2] and adopting the general formula for the  $t\bar{t}$  distribution  $d\sigma(s_+, s_-)/d\Omega_t$  found by Brzezinski *et al.*[3], one obtains the following result for the distribution:

$$\frac{d^2\sigma}{dx_f d \cos \theta_f} = \frac{3\pi\beta\alpha_{\text{EM}}^2}{2s} B_f \left[ \Theta_0^f(x_f) + \cos \theta_f \Theta_1^f(x_f) + \cos^2 \theta_f \Theta_2^f(x_f) \right], \quad (3)$$

where  $\beta$  is the top velocity,  $\alpha_{\text{EM}}$  is the fine structure constant and  $B_f$  denotes the appropriate branching fraction. The energy dependence is specified by the functions  $\Theta_i^f(x_f)$ , explicit forms of which could be found in a recent paper by the authors of this contribution[4]. They are parameterized both by production and decay form factors.

The angular distribution  $d\sigma/d \cos \theta_f$  for  $f$  could be easily obtained from eq.(3) by the integration over the energy of  $f$ . *It turns out that for  $f = \ell$  all the non-standard contributions from the decay vertex disappear upon integration over the energy  $x_f$ .* The fact that the angular leptonic distribution is insensitive to corrections to the  $V-A$  structure of the decay vertex allows for much more clear tests of the production vertices through a measurement of the distribution, since that way we can avoid a contamination from non-standard structure of the decay vertex. As an illustration, we define a  $CP$ -violating asymmetry which could be constructed using the angular distributions of  $f$  and  $\bar{f}$ :

$$\mathcal{A}_{CP}(\theta_f) = \left[ \frac{d\sigma^+(\theta_f)}{d \cos \theta_f} - \frac{d\sigma^-(\pi - \theta_f)}{d \cos \theta_f} \right] / \left[ \frac{d\sigma^+(\theta_f)}{d \cos \theta_f} + \frac{d\sigma^-(\pi - \theta_f)}{d \cos \theta_f} \right], \quad (4)$$

where  $d\sigma^{+/-}$  is referring to  $f$  and  $\bar{f}$  distributions, respectively. The asymmetry for  $f = \ell$  is sensitive to  $CP$  violation originating exclusively from the production mechanism, i.e.  $CP$ -violating form factors  $\delta D_\gamma$  and  $\delta D_Z$  while the decay vertex enters with the SM  $CP$ -conserving coupling. For bottom quarks the effect of the modification of the decay vertex is contained in corrections

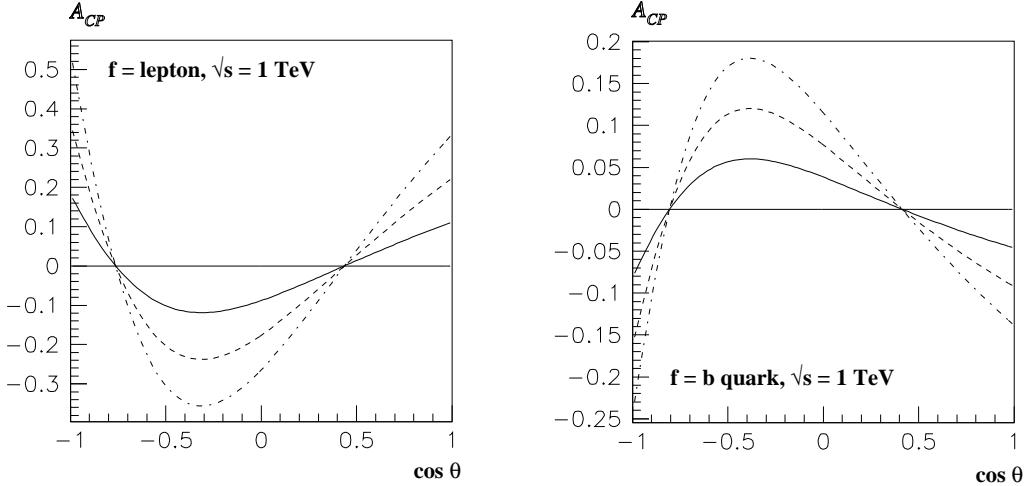


Figure 1: The  $CP$ -violating asymmetry  $A_{CP}(\theta_f)$  defined in eq.(4) as a function of  $\cos \theta_f$  for leptonic and  $b$ -quark distributions for  $\text{Re}(\delta D_\gamma) = \text{Re}(\delta D_Z) = \text{Re}(f_2^R - \bar{f}_2^L) = 0.1$  (solid line), 0.2 (dashed line), 0.3 (dash-dotted line) at  $\sqrt{s} = 1 \text{ TeV}$  collider energy.

to so called depolarization factors[4],  $\alpha^b + \bar{\alpha}^{\bar{b}} \sim \text{Re}(f_2^R - \bar{f}_2^L)$  with SM  $CP$ -conserving contribution from the production process. As it is seen from fig.1 for  $\sqrt{s} = 1 \text{ TeV}$  the asymmetry could be quite large, e.g., reaching for the semileptonic decays  $\sim 20\%$  for  $\text{Re}(\delta D_\gamma) = \text{Re}(\delta D_Z) = 0.2$ .

### 3 Optimal Observable Analysis

Using the double angular and energy distributions an expected statistical uncertainty for determination of real parts for all the form factors has been found adopting optimal observables[5] and varying the beam polarizations  $P_{e^-}$  and  $P_{e^+}$ .  $|\cos \theta| \leq 0.9$  has been assumed as a cut for the polar angle. For  $t\bar{t}$  tagging efficiency in  $\ell + 4$  jet channel we adopted 60% and for the integrated luminosity we chose the TESLA design with  $L = 500 \text{ fb}^{-1}$  at  $\sqrt{s} = 500 \text{ GeV}$ .

Generically we have observed that positive polarization led to smaller statistical errors for the eight form factors in the production vertices. For each form factor we have adjusted the optimal beam polarization such that

the statistical error was minimal:

$$\begin{aligned}
\Delta[\operatorname{Re}(\delta A_\gamma)] &= 0.16 & \text{for } P_{e^-} = 0.7 \text{ and } P_{e^+} = 0.7 \\
\Delta[\operatorname{Re}(\delta A_Z)] &= 0.07 & \text{for } P_{e^-} = 0.5 \text{ and } P_{e^+} = 0.4 \\
\Delta[\operatorname{Re}(\delta B_\gamma)] &= 0.09 & \text{for } P_{e^-} = 0.2 \text{ and } P_{e^+} = 0.2 \\
\Delta[\operatorname{Re}(\delta B_Z)] &= 0.27 & \text{for } P_{e^-} = 0.4 \text{ and } P_{e^+} = 0.4 \\
\Delta[\operatorname{Re}(\delta C_\gamma)] &= 0.11 & \text{for } P_{e^-} = 0.1 \text{ and } P_{e^+} = 0.0 \\
\Delta[\operatorname{Re}(\delta C_Z)] &= 1.11 & \text{for } P_{e^-} = 0.1 \text{ and } P_{e^+} = 0.0 \\
\Delta[\operatorname{Re}(\delta D_\gamma)] &= 0.08 & \text{for } P_{e^-} = 0.2 \text{ and } P_{e^+} = 0.1 \\
\Delta[\operatorname{Re}(\delta D_Z)] &= 14.4 & \text{for } P_{e^-} = 0.2 \text{ and } P_{e^+} = 0.1
\end{aligned} \tag{5}$$

As it is seen the precision of  $\delta\{C, D\}_Z$  measurement would be very poor even for the optimal polarization. In addition, determination of  $\delta D_\gamma$  would be difficult as well since its error varies rapidly with the polarization. For example,  $\Delta[\operatorname{Re}(\delta D_\gamma)]$  becomes 0.86 for  $P_{e^-} = 0.1/P_{e^+} = 0.1$  and 0.99 for  $P_{e^-} = 0.3/P_{e^+} = 0.1$ . The source of that sensitivity is hidden in the neutral current structure with  $\sin^2 \theta_W \simeq 0.23$ . Indeed, the optimal polarization becomes  $P_{e^-} = 0.1$  instead of 0.2 ( $\Delta[\operatorname{Re}(\delta D_\gamma)] = 0.09$ ) for  $\sin^2 \theta_W = 0.25$ . On the other hand, a good determination (almost independently of the polarization) could be expected for  $f_2^R$ . Indeed, the best precision is

$$\Delta[\operatorname{Re}(f_2^R)] = 0.01 \quad \text{for } P_{e^-} = -0.8 \text{ and } P_{e^+} = -0.8 \tag{6}$$

however, we have  $\Delta[\operatorname{Re}(f_2^R)] = 0.03$  even for  $P_{e^-} = P_{e^+} = 0$ .

## 4 Summary and Conclusions

We have presented here the angular and energy distributions for  $f^{(-)}$  in the process  $e^+ e^- \rightarrow t\bar{t} \rightarrow f^{(-)} \dots$ , where  $f = \ell$  or  $b$  quark. The most general ( $CP$ -violating and  $CP$ -conserving) couplings for  $\gamma t\bar{t}$ ,  $Z t\bar{t}$  and  $W tb$  have been assumed. The bottom-quark mass has been neglected and we have kept only terms linear in anomalous couplings.

Test of  $CP$  violation has also been discussed, introducing a  $CP$ -sensitive asymmetry  $\mathcal{A}_{CP}$  as an example.

Using the double angular and energy distribution of a lepton we have found that at  $\sqrt{s} = 500$  GeV with the integrated luminosity  $L = 500 \text{ fb}^{-1}$  the best determined top-quark coupling would be the axial coupling of the

Z boson with the error  $\Delta[\text{Re}(\delta A_Z)] = 0.07$  while the lowest precision is expected for  $\text{Re}(\delta D_Z)$  with  $\Delta[\text{Re}(\delta D_Z)] = 14.4$ .

## Note added

After this work has been presented, a paper by Rindani[6] appeared where the double angular and energy leptonic distribution has been also found.

## Acknowledgments

BG is grateful to the organizers of the PASCOS99 conference for creating a very warm and inspiring atmosphere during the meeting. This work is supported in part by the State Committee for Scientific Research (Poland) under grant 2 P03B 014 14 and by Maria Skłodowska-Curie Joint Fund II (Poland-USA) under grant MEN/NSF-96-252.

## References

- [1] CDF Collaboration : F. Abe et al., *Phys. Rev. Lett.* **73** (225), 1994;  
*Phys. Rev.* **D50** (2966), 1994; *Phys. Rev. Lett.* **74** (2626), 1995;  
D0 Collaboration : S. Abachi et al., *Phys. Rev. Lett.* **74** (2632), 1995.
- [2] Y.S. Tsai, *Phys. Rev.* **D4** (2821), 1971;  
S. Kawasaki, T. Shirafuji and S.Y. Tsai, *Prog. Theor. Phys.* **49**, 1656 (1973).
- [3] L. Brzeziński, B. Grzadkowski and Z. Hioki, *Int. J. Mod. Phys.* **14**, 1261 (1999), hep-ph/9710358.
- [4] B. Grzadkowski and Z. Hioki, *Phys. Lett.* **B476** (87), 2000, hep-ph/9911505.
- [5] J.F. Gunion, B. Grzadkowski and X-G. He, *Phys. Rev. Lett.* **77** (5172), 1996, hep-ph/9605326.  
See also:  
D. Atwood and A. Soni, *Phys. Rev.* **D45** (2405), 1992;

M. Davier, L. Duflot, F. Le Diberder and A. Roug , *Phys. Lett.* **B306** (411), 1993;  
M. Diehl and O. Nachtmann, *Z. Phys.* **C62**, 397 (1994);  
B. Grzadkowski and J.F. Gunion, *Phys. Lett.* **B350** (218), 1995, hep-ph/9501339.

[6] S. Rindani, hep-ph/0002006.